

# Q U A N T U M   L A T T I C E   M O D E L

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## Geometric Routing and Gravitational Throttling Exterior Phase–Action Routing and the Schwarzschild Throttle

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February 25, 2026

Updated: June 14, 2026

### Abstract

This paper develops the exterior routing-admittance sector of the Quantum Lattice Model (QLM) by deriving gravitational throttling from geometric suppression of radial phase–action transport. Building on the primitive lattice structure

$$\{\hbar, \ell_P, t_P\}, \quad c = \frac{\ell_P}{t_P}, \quad m_P = \frac{\hbar t_P}{\ell_P^2},$$

the analysis introduces a dimensionless scalar routing availability

$$Y \equiv \frac{Y_{\text{eff}}}{Y_P},$$

defined as the fraction of vacuum routing capacity available to phase–action transport.

For a spherically symmetric mass  $M$ , the mass–energy content defines a dimensionless closure inventory

$$N_M \equiv \frac{Mc^2 t_P}{\hbar} = \frac{M}{m_P}.$$

Radial phase–action transport is supported by transverse spherical shells of area  $4\pi r^2$ . Because the shell has two independent transverse routing directions, the local radial routing decrement obeys

$$dY = -2N_M \frac{\ell_P}{r^2} dr.$$

With the boundary condition  $Y(\infty) = 1$ , this integrates to the exterior routing field

$$Y(r) = 1 - 2 \left( \frac{M}{m_P} \right) \frac{\ell_P}{r}.$$

The vanishing of  $Y(r)$  defines the Schwarzschild critical radius

$$r_s = 2 \left( \frac{M}{m_P} \right) \ell_P,$$

which is equivalent to the standard form  $r_s = 2GM/c^2$  when

$$G = \frac{\ell_P^5}{t_P^3 \hbar}.$$

The gravitational redshift factor is then interpreted as a routing throttle,

$$Z_g(r) = Y(r)^{-1/2} = \left(1 - \frac{r_s}{r}\right)^{-1/2},$$

so that local phase–action evolution remains proper-time invariant while outward transport relative to infinity is asymptotically suppressed.

The finite interior endpoint of collapse is not rederived here. It is provided by the density-cap result of *Planck Energy Density Cap and Minimal Saturated-Core Completion*, which supplies the local bound  $u(r) \leq u_P$  and the corresponding saturated-core radius. Thus the present work supplies the exterior routing derivation of gravitational throttling, while the density-cap construction supplies the compatible non-singular interior completion. No new primitives or free parameters are introduced.

## 1 Introduction

As established in [8], Lorentz kinematics and the invariant bridge

$$\frac{E}{p} = c \tag{1}$$

emerge directly from the per-radian Planck primitives

$$\{\hbar, \ell_P, t_P\}, \quad c = \frac{\ell_P}{t_P}, \tag{2}$$

and invariant proper-tick accumulation. With the kinematic structure fixed by lattice transport, the next structural question is how phase–action flow is routed when the available transport channels are geometrically restricted.

The present work develops the exterior gravitational routing sector of the Quantum Lattice Model. Its purpose is to derive the scalar radial routing field associated with a spherically symmetric mass distribution and to show that the resulting throttling law reproduces the Schwarzschild critical radius and gravitational redshift factor. The comparison target is the standard exterior Schwarzschild form of general relativity [2, 3, 4, 5]. The interior endpoint of collapse is not rederived here; it is supplied by the local density-cap construction developed in *Planck Energy Density Cap and Minimal Saturated-Core Completion* [7].

The analysis proceeds in four steps.

1. A dimensionless scalar routing availability

$$Y \equiv \frac{Y_{\text{eff}}}{Y_P} \tag{3}$$

is introduced as the normalized fraction of vacuum routing capacity available to phase–action transport. This scalar field measures exterior radial routing availability; tensor routing structures are reserved for later extensions.

2. The mass-energy content of a spherically symmetric source is expressed as a dimensionless closure inventory,

$$N_M \equiv \frac{Mc^2 t_P}{\hbar} = \frac{M}{m_P}, \quad m_P = \frac{\hbar t_P}{\ell_P^2}. \tag{4}$$

This quantity provides the source strength for radial routing suppression.

3. The transverse shell geometry supporting outward radial transport is analyzed. Since a spherical shell has two independent transverse routing directions, the local routing decrement takes the form

$$dY = -2N_M \frac{\ell_P}{r^2} dr. \quad (5)$$

With the boundary condition

$$Y(\infty) = 1, \quad (6)$$

this integrates to

$$Y(r) = 1 - 2 \left( \frac{M}{m_P} \right) \frac{\ell_P}{r}. \quad (7)$$

4. The zero of  $Y(r)$  defines the Schwarzschild critical radius,

$$r_s = 2 \left( \frac{M}{m_P} \right) \ell_P, \quad (8)$$

and comparison with

$$r_s = \frac{2GM}{c^2} \quad (9)$$

yields the QLM gravitational coupling

$$G = \frac{\ell_P^5}{t_P^3 \hbar}. \quad (10)$$

The associated redshift throttle is

$$Z_g(r) = Y(r)^{-1/2} = \left( 1 - \frac{r_s}{r} \right)^{-1/2}. \quad (11)$$

The finite saturated-core radius and local energy-density cap are used only as previously established interior-completion results. In this way the present paper remains focused on the exterior routing derivation: gravity is interpreted as asymptotic throttling of radial phase-action transport, while the non-singular interior completion is supplied by the density-cap result of [7].

No new primitives or free parameters are introduced. All gravitational quantities derived below reduce algebraically to the primitive lattice set

$$\{\hbar, \ell_P, t_P\}. \quad (12)$$

## 2 Canonical Primitives and Impedance Normalization

QLM adopts the Planck primitive set [1]

$$\{\hbar, \ell_P, t_P\}, \quad c = \frac{\ell_P}{t_P}. \quad (13)$$

The corresponding QLM Planck mass is

$$m_P = \frac{\hbar t_P}{\ell_P^2}. \quad (14)$$

This quantity will be used below to express the mass-energy content of a source in dimensionless Planck closure units.

Impedance in QLM is defined operationally as the ratio of transverse action flux to phase-gradient response in the null limit. Because phase-action transport is fundamental, impedance normalization must be expressed in terms of lattice primitives and the vacuum routing baseline.

In the vacuum, unsuppressed phase-action transport propagates at the invariant null speed  $c$ . The corresponding reference impedance scale is therefore associated with full lattice routing availability: no geometric restriction reduces the transverse action flux available to support outward propagation.

For routing comparisons it is useful to introduce a geometrically unwrapped normalization that removes the  $4\pi$  spherical wrapping factor associated with isotropic emission. This isolates the local routing capacity from the global angular wrapping of a spherical wave.

Define the Planck-normalized impedance and admittance by

$$Z_P \equiv \frac{Z_0}{4\pi}, \quad Y_P \equiv \frac{1}{Z_P}. \quad (15)$$

Here  $Z_0$  denotes the vacuum wave impedance associated with unsuppressed null transport, while  $Y_P$  sets the reference full-availability routing baseline of the lattice.

Effective routing admittances will be defined below as dimensionless ratios relative to this Planck-normalized baseline, so that geometric suppression factors depend on routing geometry rather than on the introduction of additional microscopic constants.

### 3 Routing Availability and Closure Inventory

#### 3.1 Scalar Routing Availability

Let  $Y_{\text{eff}}$  denote the effective routing admittance associated with an allowed phase-action transport configuration. Here admittance refers to normalized phase-action transport capacity, not electromagnetic material response. Boundary geometry modifies the effective routing multiplicity without altering the primitive lattice constants; suppression arises from geometric restriction of admissible transport channels.

For the exterior gravitational sector developed in this paper, the relevant transport is open radial transport. Phase-action flow propagates along the outward radial direction, while the transverse spherical shell determines the available routing support at radius  $r$ . The effective admittance therefore measures the fraction of vacuum radial routing capacity available at that shell.

Define the dimensionless scalar routing availability

$$Y \equiv \frac{Y_{\text{eff}}}{Y_P} = \frac{Z_P}{Z_{\text{eff}}}. \quad (16)$$

The quantity  $Y$  is dimensionless and purely geometric. It is not a kinematic velocity parameter and does not represent mass. Instead, it encodes the availability of admissible routing channels relative to full vacuum lattice availability.

The limiting cases are

$$Y = 1 \quad \text{full vacuum routing availability,} \quad (17)$$

$$0 < Y < 1 \quad \text{suppressed radial routing availability,} \quad (18)$$

and

$$Y = 0 \quad \text{complete asymptotic radial throttling.} \quad (19)$$

In the spherically symmetric exterior problem, the scalar routing availability becomes a radial field  $Y(r)$ . The symbol  $Y(r)$  is reserved here for scalar exterior routing availability. General tensor routing structures, when required, should be denoted by  $\mathcal{Y}^{ij}(x)$  and treated separately.

### 3.2 Closure Inventory as Source Strength

The scalar routing field derived in this work concerns open radial transport. The source of the exterior routing field is therefore not the detailed internal closure geometry of the mass distribution, but its total mass-energy content.

For a spherically symmetric source of total mass  $M$ , the rest-energy content is

$$E_M = Mc^2. \quad (20)$$

Over one Planck tick, this corresponds to the action throughput

$$S_M = E_M t_P = Mc^2 t_P. \quad (21)$$

Normalizing by the reduced-action quantum gives the dimensionless closure inventory

$$N_M \equiv \frac{S_M}{\hbar} = \frac{Mc^2 t_P}{\hbar}. \quad (22)$$

Using the primitive identities

$$c = \frac{\ell_P}{t_P}, \quad m_P = \frac{\hbar t_P}{\ell_P^2}, \quad (23)$$

the closure inventory reduces to

$$N_M = \frac{M}{m_P}. \quad (24)$$

Thus  $N_M$  counts the source mass-energy in Planck closure units and provides the dimensionless source strength for exterior radial routing suppression. The next section derives how this source strength is distributed over the transverse shell geometry supporting outward phase-action transport.

### 3.3 Radial Shell Routing Geometry

The exterior gravitational routing problem concerns open radial phase-action transport. For a spherically symmetric source, outward transport occurs along the radial direction  $\hat{r}$ , while the available routing support is supplied by the transverse spherical shell orthogonal to that direction.

At radius  $r$ , the transverse shell has area

$$A(r) = 4\pi r^2. \quad (25)$$

If the elementary transverse routing patch is Planck-scale with area  $\ell_P^2$ , the corresponding number of available shell routing channels is approximately

$$N_{\text{shell}}(r) \sim \frac{4\pi r^2}{\ell_P^2}. \quad (26)$$

This shell support represents the vacuum routing capacity available to outward radial phase-action transport at radius  $r$ . Since the shell area grows as  $r^2$ , any fixed source strength distributed over this support must dilute with inverse-square scaling.

The radial dependence of the scalar routing availability is therefore controlled by how the source strength is distributed over this transverse shell support.

The task is then to determine how the closure inventory derived above reduces the available radial routing capacity as it is distributed over the transverse shell geometry.

Because the shell area scales as  $A(r) \propto r^2$ , spherical symmetry requires the local routing decrement to scale as

$$dY \propto -N_M \frac{\ell_P}{r^2} dr. \quad (27)$$

The remaining factor is fixed by the two independent transverse routing directions of the spherical shell. This gives the shell-routing balance law derived in the next section.

## 4 Exterior Routing Field

### 4.1 Shell-Routing Balance Law

The preceding section established the three ingredients required for the exterior routing field: the scalar routing availability  $Y(r)$ , the closure inventory  $N_M$ , and the transverse shell support  $A(r) = 4\pi r^2$ . The remaining task is to determine the local decrement in radial routing availability produced by the source strength  $N_M$ .

Because the available shell support grows as  $r^2$ , spherical symmetry requires the routing decrement per radial increment to dilute as  $r^{-2}$ . For a single transverse routing direction, the differential suppression takes the form

$$dY_{\perp} = -N_M \frac{\ell_P}{r^2} dr. \quad (28)$$

The factor  $\ell_P/r^2$  has dimension 1/length, so  $dY_{\perp}$  is dimensionless as required. The minus sign indicates that the available routing capacity decreases as the field is evaluated inward toward the source.

The spherical shell has two independent transverse routing directions,  $(\theta, \phi)$ . The total routing decrement is therefore the transverse sum

$$dY = 2 dY_{\perp}. \quad (29)$$

Substituting Eq. (28) gives the shell-routing balance law

$$dY = -2N_M \frac{\ell_P}{r^2} dr. \quad (30)$$

This relation is the local exterior routing law for a spherically symmetric source. It expresses the inverse-square dilution of closure inventory over the transverse shell support, with the factor of two fixed by the two transverse directions of the spherical shell.

### 4.2 Geometric Origin of the Factor Two

The factor of 2 in Eq. (30) arises from the dimensionality of the transverse shell manifold and can be seen in two equivalent ways.

First, for the radial unit vector  $\hat{r} = \mathbf{r}/r$ , the divergence in three spatial dimensions is

$$\nabla \cdot \hat{r} = \frac{2}{r}. \quad (31)$$

This follows from the general radial divergence formula

$$\nabla \cdot [f(r)\hat{r}] = \frac{1}{r^2} \frac{d}{dr} (r^2 f(r)), \quad (32)$$

which for  $f(r) = 1$  gives

$$\nabla \cdot \hat{r} = \frac{1}{r^2} \frac{d}{dr}(r^2) = \frac{2r}{r^2} = \frac{2}{r}. \quad (33)$$

The factor 2 therefore reflects the differential growth of the spherical shell area  $A(r) = 4\pi r^2$  with radius.

Second, the shell has two independent transverse directions,  $(\theta, \phi)$ . Under spherical symmetry, the local transverse suppression tensor on the shell may be written

$$\Sigma_{ab}(r) = \sigma(r)\delta_{ab}, \quad a, b \in \{\theta, \phi\}. \quad (34)$$

The total transverse suppression is the trace

$$\text{Tr}_\perp(\Sigma) = \lambda_\theta + \lambda_\phi = 2\sigma(r). \quad (35)$$

Equations (31)–(35) show that the factor of two is geometric rather than adjustable. It encodes the two-dimensional multiplicity of the transverse routing surface and therefore introduces no new free coefficient.

### 4.3 Integration of the Routing Field

The vacuum boundary condition is

$$Y(\infty) = 1, \quad (36)$$

corresponding to full radial routing availability far from the source.

Integrating Eq. (30) from infinity to radius  $r$  gives

$$Y(r) = 1 - 2N_M \frac{\ell_P}{r}. \quad (37)$$

Using the closure inventory relation  $N_M = M/m_P$ , this becomes

$$Y(r) = 1 - 2 \left( \frac{M}{m_P} \right) \frac{\ell_P}{r}. \quad (38)$$

Thus exterior radial routing availability decreases with the source mass-energy content and approaches the vacuum value  $Y = 1$  as  $r \rightarrow \infty$ .

### 4.4 Critical Routing Radius

The critical routing radius occurs where radial routing availability vanishes,

$$Y(r_s) = 0. \quad (39)$$

Substituting Eq. (38) gives

$$r_s = 2 \left( \frac{M}{m_P} \right) \ell_P. \quad (40)$$

This is the Schwarzschild critical radius expressed in QLM primitive form. At this radius, outward radial routing is completely throttled relative to an asymptotic observer.

## 4.5 Emergence of the Gravitational Constant

In standard relativistic notation, the Schwarzschild radius is

$$r_s = \frac{2GM}{c^2}. \quad (41)$$

Equating Eq. (40) with Eq. (41) yields

$$G = \frac{c^2 \ell_P}{m_P}. \quad (42)$$

Using

$$c = \frac{\ell_P}{t_P}, \quad m_P = \frac{\hbar t_P}{\ell_P^2}, \quad (43)$$

Eq. (42) reduces to

$$\begin{aligned} G &= \frac{(\ell_P/t_P)^2 \ell_P}{\hbar t_P / \ell_P^2} \\ &= \frac{\ell_P^5}{t_P^3 \hbar}. \end{aligned} \quad (44)$$

Therefore,

$$G = \frac{\ell_P^5}{t_P^3 \hbar}. \quad (45)$$

The gravitational constant is therefore not introduced as an independent primitive in this construction. It appears as the derived lattice coupling required for the routing critical radius (40) to match the Schwarzschild form (41).

## 5 Gravitational Throttling

Lorentz kinematics derived in [8] establish invariant lattice transport from per-radian Planck primitives and proper-tick accumulation. In the routing framework developed above, the exterior gravitational field arises from suppressed radial phase-action transport characterized by the scalar routing availability field  $Y(r)$ .

Because local lattice updates remain fixed by the primitive action increment  $\hbar$  per tick, gravitational effects appear through the mapping between local proper-time updates and the rates measured by an observer at infinity. In the routing interpretation this mapping is controlled by the available fraction of radial routing capacity.

The scalar field  $Y(r)$  represents an intensity-like transport capacity. The corresponding amplitude-level proper-time throttle is therefore given by the square root of the routing availability,

$$\frac{d\tau}{dt_\infty} = \sqrt{Y(r)}. \quad (46)$$

Substituting the exterior routing field

$$Y(r) = 1 - 2 \left( \frac{M}{m_P} \right) \frac{\ell_P}{r} \quad (47)$$

and using

$$r_s = 2 \left( \frac{M}{m_P} \right) \ell_P, \quad (48)$$

gives

$$\frac{d\tau}{dt_\infty} = \sqrt{1 - \frac{r_s}{r}}. \quad (49)$$

The inverse factor defines the gravitational routing throttle,

$$Z_g(r) \equiv \left(\frac{d\tau}{dt_\infty}\right)^{-1} = Y(r)^{-1/2} = \left(1 - \frac{r_s}{r}\right)^{-1/2}. \quad (50)$$

The quantity  $Z_g(r)$  is not an additional gravitational field or a modification of general relativity. It is the standard Schwarzschild redshift factor written as an impedance-like throttle on asymptotic phase-action transport.

Local action transport remains fixed by the lattice update rule, while the rate mapped to infinity is reduced by the throttle factor:

$$\left(\frac{dS}{dt}\right)_\infty = \frac{1}{Z_g(r)} \left(\frac{dS}{d\tau}\right). \quad (51)$$

Thus the exterior routing field  $Y(r)$  determines both the Schwarzschild critical radius and the gravitational redshift throttle. The interior completion remains supplied by the density-cap construction referenced above; it is not rederived in this section.

## 6 Relation to the Density-Cap Interior Completion

The exterior routing field derived above determines the radial phase-action throttle outside a spherically symmetric source. It does not, by itself, determine the final interior state reached during gravitational collapse. The local interior completion is supplied by the density-cap construction developed in *Planck Energy Density Cap and Minimal Saturated-Core Completion* [7].

That work establishes the local QLM energy-density bound

$$u(r) \leq u_P, \quad u_P = \frac{\hbar}{\ell_P^3 t_P}, \quad (52)$$

together with the corresponding maximum mass density

$$\rho_{\max} = \frac{m_P}{\ell_P^3}. \quad (53)$$

For a spherically symmetric saturated interior, the resulting core radius is

$$R_c(M) = \left(\frac{3}{4\pi} \frac{M}{m_P}\right)^{1/3} \ell_P. \quad (54)$$

The role of the present paper is therefore complementary. The routing field  $Y(r)$  and throttle  $Z_g(r)$  describe the exterior Schwarzschild-compatible routing structure, while the density-cap result supplies the finite non-singular interior completion.

## 7 Conclusion

This paper developed the exterior gravitational routing sector of the Quantum Lattice Model by interpreting gravity as geometric suppression of outward radial phase-action transport. Starting from the primitive lattice set

$$\{\hbar, \ell_P, t_P\}, \quad c = \frac{\ell_P}{t_P}, \quad (55)$$

and the QLM Planck mass

$$m_P = \frac{\hbar t_P}{\ell_P^2}, \quad (56)$$

the mass-energy content of a spherically symmetric source was expressed as the dimensionless closure inventory

$$N_M = \frac{M}{m_P}. \quad (57)$$

The exterior routing problem was then reduced to the distribution of this closure inventory over the transverse spherical shell supporting radial phase-action transport. Because the shell support scales as  $4\pi r^2$  and possesses two independent transverse routing directions, the local radial routing decrement was found to be

$$dY = -2N_M \frac{\ell_P}{r^2} dr. \quad (58)$$

With the boundary condition

$$Y(\infty) = 1, \quad (59)$$

this integrates to

$$Y(r) = 1 - 2 \left( \frac{M}{m_P} \right) \frac{\ell_P}{r}. \quad (60)$$

The zero of this scalar routing availability defines the critical routing radius

$$r_s = 2 \left( \frac{M}{m_P} \right) \ell_P, \quad (61)$$

which is equivalent to the Schwarzschild radius

$$r_s = \frac{2GM}{c^2} \quad (62)$$

when the gravitational coupling is written as the derived lattice quantity

$$G = \frac{\ell_P^5}{t_P^3 \hbar}. \quad (63)$$

Thus  $G$  is not introduced as an independent primitive in this construction; it emerges from the primitive lattice scales and the exterior routing geometry.

The corresponding gravitational throttle is

$$Z_g(r) = Y(r)^{-1/2} = \left( 1 - \frac{r_s}{r} \right)^{-1/2}, \quad (64)$$

which is the standard Schwarzschild redshift factor written as an impedance-like throttle on asymptotic phase-action transport. Local phase-action evolution remains governed by proper-tick accumulation, while the rate mapped to infinity is reduced by the routing throttle.

The finite interior endpoint of collapse is not rederived in this paper. It is supplied by the density-cap construction of *Planck Energy Density Cap and Minimal Saturated-Core Completion*, which provides the local bound

$$u(r) \leq u_P \tag{65}$$

and the corresponding saturated-core radius

$$R_c(M) = \left( \frac{3}{4\pi} \frac{M}{m_P} \right)^{1/3} \ell_P. \tag{66}$$

No additional primitives, free parameters, or modifications to the exterior Schwarzschild form are introduced. Within QLM, gravity is therefore interpreted as asymptotic throttling of radial phase-action transport produced by geometric suppression of exterior routing availability.

# Notation and Symbol Definitions

Symbol / Notation	Definition
<i>Framework and conventions</i>	
QLM	Quantum Lattice Model.
GR	General relativity.
primitive	Quantity taken as foundational in the QLM construction rather than derived from other quantities.
derived	Quantity obtained algebraically from primitive quantities and identities.
per-radian	Quantity normalized to one radian of phase advance.
null	Propagation saturating the invariant lattice transport bound $c = \ell_P/t_P$ .
routing	Geometric availability of admissible phase-action transport channels.
throttling	Reduction of asymptotically measured phase-action transport relative to local proper-time evolution.
<i>Primitive QLM lattice quantities</i>	
$\hbar$	Reduced Planck constant; primitive reduced-action quantum of the QLM.
$\ell_P$	Planck length; primitive lattice spatial increment.
$t_P$	Planck time; primitive lattice temporal increment, or one lattice tick.
$c$	Invariant lattice transport speed, $c = \ell_P/t_P$ .
<i>Planck-scale derived quantities</i>	
$E_P$	Planck energy, $E_P = \hbar/t_P$ .
$p_P$	Planck momentum, $p_P = \hbar/\ell_P$ .
$m_P$	QLM Planck mass, $m_P = \hbar t_P/\ell_P^2$ .
$u_P$	Planck energy density, $u_P = \hbar/(\ell_P^3 t_P)$ .
$\rho_{\max}$	Maximum lattice mass density, $\rho_{\max} = m_P/\ell_P^3$ .
<i>Phase-action quantities</i>	
$S$	Action.
$dS$	Infinitesimal action increment.
$\Delta S$	Finite action increment; for one saturated lattice tick, $\Delta S = \hbar$ .
$E$	Energy, $E = \hbar\omega$ .
$p_i$	Momentum component, $p_i = \hbar k_i$ .
$\omega$	Temporal phase rate.
$k_i$	Spatial phase-gradient component.
<i>Impedance and routing quantities</i>	
$Z_0$	Vacuum wave impedance associated with unsuppressed null transport.
$Z_P$	Per-radian Planck-normalized impedance, $Z_P = Z_0/(4\pi)$ .
$Y_P$	Per-radian Planck-normalized admittance, $Y_P = 1/Z_P$ .
$Y_{\text{eff}}$	Effective routing admittance associated with an allowed transport configuration.

Symbol / Notation	Definition
$Z_{\text{eff}}$	Effective routing impedance associated with an allowed transport configuration.
$Y$	Dimensionless scalar routing availability, $Y = Y_{\text{eff}}/Y_P = Z_P/Z_{\text{eff}}$ .
$Y(r)$	Exterior radial routing-availability field; fraction of vacuum routing capacity available for outward radial transport at radius $r$ .
$\mathcal{Y}^{ij}(x)$	General tensor routing structure reserved for later extensions; not identical to the scalar field $Y(r)$ .
<i>Radial shell-routing quantities</i>	
$r$	Exterior radial coordinate.
$\hat{r}$	Radial unit vector.
$A(r)$	Spherical shell area at radius $r$ , $A(r) = 4\pi r^2$ .
$N_{\text{shell}}(r)$	Approximate number of Planck-scale shell routing channels at radius $r$ .
$\theta, \phi$	Angular coordinates labeling the two transverse directions on the spherical shell.
$\Sigma_{ab}(r)$	Local transverse suppression tensor on the spherical shell.
$\text{Tr}_{\perp}(\Sigma)$	Transverse trace of the shell suppression tensor.
<i>Gravitational source and exterior field quantities</i>	
$M$	Total gravitating mass of the spherically symmetric source.
$S_M$	Action throughput associated with rest-energy $Mc^2$ over one Planck tick, $S_M = Mc^2 t_P$ .
$N_M$	Dimensionless closure inventory associated with mass $M$ , $N_M = Mc^2 t_P / \hbar = M/m_P$ .
$G$	Newtonian gravitational constant; in QLM primitive form, $G = \ell_P^5 / (t_P^3 \hbar)$ .
$r_s$	Schwarzschild critical radius, $r_s = 2(M/m_P)\ell_P = 2GM/c^2$ .
$Z_g(r)$	Gravitational throttle / Schwarzschild redshift factor, $Z_g(r) = Y(r)^{-1/2} = (1 - r_s/r)^{-1/2}$ .
$\tau$	Local proper time.
$t_{\infty}$	Asymptotic coordinate time measured by an observer at infinity.
<i>Imported density-cap quantities</i>	
$u(r)$	Local proper energy density.
$R_c(M)$	Saturated-core radius imported from the density-cap completion, $R_c(M) = \left(\frac{3}{4\pi} \frac{M}{m_P}\right)^{1/3} \ell_P$ .

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