Mass as Temporal–Spatial Phase Closure

Recovering $E = mc^2$ from $m = \hbar\tau/r^2$

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Abstract

This paper develops a focused Quantum Lattice Model (QLM) interpretation of rest mass as local temporal–spatial phase-action closure. Starting from the QLM primitive triplet

$$\{\hbar, \ell_P, t_P\}, \qquad c = \frac{\ell_P}{t_P},$$

and the proper-time phase-flow law

$$E = \hbar\frac{d\theta}{d\tau},$$

we show that the Planck mass relation

$$m_P = \frac{\hbar t_P}{\ell_P^2}$$

is the saturated endpoint of a more general closure form

$$m = \frac{\hbar\tau}{r^2}.$$

When the closure satisfies the QLM transport condition

$$r = c\tau,$$

this expression reduces directly to

$$mc^2 = \hbar\omega, \qquad \omega = \frac{1}{\tau}.$$

Thus the inertial energy relation is recovered from temporal–spatial closure rather than treated as an independent starting point. The Planck mode corresponds to

$$\tau = t_P, \qquad r = \ell_P, \qquad m = m_P,$$

while the electron rest scale corresponds to

$$\tau_e = \frac{\hbar}{m_e c^2}, \qquad r_e = \bar{\lambda}_e = \frac{\hbar}{m_e c}, \qquad r_e = c\tau_e.$$

The electron-to-Planck ratio is therefore

$$\epsilon_e \equiv \frac{m_e}{m_P} = \frac{t_P}{\tau_e} = \frac{\ell_P}{r_e} = \omega_e t_P.$$

The paper further distinguishes invariant local rest mass from routed momentum: gravitational, Coulomb, or geometric routing may suppress transported momentum or externally observed phase-action flow without changing the electron's intrinsic rest closure mass. The fine-structure constant is included only as a Coulomb-routing factor relating the electron rest closure scale $r_e = \bar{\lambda}_e$ to the Bohr scale $a_0 = \bar{\lambda}_e/\alpha$.

1 Introduction

The Quantum Lattice Model (QLM) formulates physics as deterministic phase–action transport on a discrete Planck-scale lattice. Its primitive triplet is

$$\{\hbar, \ell_P, t_P\}, \tag{1}$$

where $\hbar$ is reduced action per radian, ℓ_P is the primitive spatial lattice spacing, and t_P is the primitive proper-time tick [6].

The invariant causal update speed is

$$c = \frac{\ell_P}{t_P}. \tag{2}$$

The fundamental QLM phase-flow law is

$$E = \hbar \frac{d\theta}{d\tau}, \tag{3}$$

where θ is dimensionless physical phase measured in radians and τ is local proper time along a worldline [6, 7].

For a saturated Planck tick,

$$\Delta\theta = 1, \qquad \Delta\tau = t_P, \tag{4}$$

the Planck energy is

$$E_P = \frac{\hbar}{t_P}. \tag{5}$$

The corresponding Planck mass is

$$m_P = \frac{E_P}{c^2} = \frac{\hbar t_P}{\ell_P^2}. \tag{6}$$

This relation was isolated as a standalone QLM identity in Ref. [9].

The present paper shows that the same temporal–spatial structure that produces the saturated Planck mass also gives a direct route to the inertial energy relation $E = mc^2$.

The closure form developed here is

$$m = \frac{\hbar\tau}{r^2}, \tag{7}$$

where r is a local spatial closure radius and τ is the associated local proper-time closure period. When the closure satisfies

$$r = c\tau, \tag{8}$$

Eq. (7) reduces exactly to

$$mc^2 = \hbar\omega, \qquad \omega = \frac{1}{\tau}. \tag{9}$$

The Planck mode is recovered by setting

$$\tau = t_P, \qquad r = \ell_P. \tag{10}$$

The electron rest scale is recovered by setting

$$\tau_e = \frac{\hbar}{m_e c^2}, \qquad r_e = \bar{\lambda}_e = \frac{\hbar}{m_e c}. \tag{11}$$

This paper does not attempt to derive the numerical value of m_e or α. Rather, it shows that once m_e is supplied, the electron rest scale fits the same temporal–spatial closure structure as the saturated Planck mode:

$$\epsilon_e \equiv \frac{m_e}{m_P} = \omega_e t_P = \frac{t_P}{\tau_e} = \frac{\ell_P}{r_e}. \tag{12}$$

2 Scope

This paper is restricted to a single closure identity within the Quantum Lattice Model:

$$m = \frac{\hbar\tau}{r^2}, \qquad r = c\tau. \tag{13}$$

The purpose is to show that this temporal–spatial closure form reduces directly to

$$mc^2 = \hbar\omega, \tag{14}$$

and that the Planck mass and electron rest scale appear as two instances of the same closure structure.

The paper does not claim to derive the measured electron mass, the fine-structure constant, or the full charge-sector dynamics. Its narrower purpose is to establish the closure chain:

$$m_P = \frac{\hbar t_P}{\ell_P^2} \quad \longrightarrow \quad m = \frac{\hbar\tau}{r^2} \quad \xrightarrow{r=c\tau} \quad mc^2 = \hbar\omega \quad \longrightarrow \quad m_e = \frac{\hbar\tau_e}{r_e^2}. \tag{15}$$

3 Notation and Conventions

Throughout this paper, all phase rates are angular rates measured in radians per unit proper time. Thus

$$\omega = \frac{d\theta}{d\tau}, \tag{16}$$

and no factor of 2π appears in the primitive energy relation

$$E = \hbar\omega. \tag{17}$$

Cycle frequency is derived and is not used as a primitive quantity.

The symbol $\bar{\lambda}_e$ denotes the reduced Compton wavelength of the electron,

$$\bar{\lambda}_e = \frac{\hbar}{m_e c}, \tag{18}$$

not the full Compton wavelength

$$\lambda_e = \frac{h}{m_e c} = 2\pi\bar{\lambda}_e. \tag{19}$$

The symbol τ denotes a local proper-time closure period, not an asymptotic coordinate time. Routing variables such as Y, $Y_g(r)$, and $Z_g(r)$ modify transported phase-action or routed momentum, not the intrinsic rest mass m of a local closure mode.

A complete notation summary is provided in Sec. A.

4 Primitive Starting Point

The QLM primitive set is

$$\{\hbar, \ell_P, t_P\}. \tag{20}$$

The invariant transport identity is

$$c = \frac{\ell_P}{t_P}. \tag{21}$$

The fundamental phase-action relation is

$$dS = \hbar\, d\theta. \tag{22}$$

Dividing by local proper time τ gives

$$E = \frac{dS}{d\tau} = \hbar \frac{d\theta}{d\tau}. \tag{23}$$

Defining angular frequency by

$$\omega \equiv \frac{d\theta}{d\tau}, \tag{24}$$

we obtain

$$E = \hbar\omega. \tag{25}$$

For a saturated Planck tick,

$$\Delta\theta = 1, \qquad \Delta\tau = t_P, \qquad \Delta S = \hbar, \tag{26}$$

so that

$$\omega_P = \frac{1}{t_P}, \tag{27}$$

and

$$E_P = \hbar\omega_P = \frac{\hbar}{t_P}. \tag{28}$$

5 Planck Mass as the Saturated Closure Mode

The Planck energy is the saturated one-tick phase-action throughput,

$$E_P = \frac{\hbar}{t_P}. \tag{29}$$

Using the Planck Energy Identity relation established in Ref. [9],

$$E_P = m_P c^2, \tag{30}$$

the Planck mass is

$$m_P = \frac{E_P}{c^2}. \tag{31}$$

Substituting $E_P = \hbar/t_P$ and $c = \ell_P/t_P$, we obtain

$$\begin{aligned} m_P &= \frac{\hbar/t_P}{(\ell_P/t_P)^2} \\ &= \frac{\hbar t_P}{\ell_P^2}. \end{aligned} \tag{32}$$

Thus,

$$m_P = \frac{\hbar t_P}{\ell_P^2}. \tag{33}$$

The same result is recovered from the general temporal–spatial closure form

$$m = \frac{\hbar \tau}{r^2} \tag{34}$$

by setting the saturated Planck closure values

$$\tau = t_P, \qquad r = \ell_P. \tag{35}$$

Then

$$\begin{aligned} m &= \frac{\hbar t_P}{\ell_P^2} \\ &= m_P. \end{aligned} \tag{36}$$

The associated Planck angular frequency is

$$\omega_P = \frac{1}{t_P}, \tag{37}$$

so that

$$m_P c^2 = \hbar \omega_P = \frac{\hbar}{t_P} = E_P. \tag{38}$$

The Planck mass is therefore the saturated temporal–spatial closure mode: one Planck tick, one Planck length, and one Planck energy of phase-action throughput.

6 Dimensional Meaning of $\hbar\tau/r^2$

Consider the quantity

$$\frac{\hbar \tau}{r^2}, \tag{39}$$

where τ is a local proper-time scale and r is a local spatial closure scale.

The dimensions of $\hbar\tau$ are

$$[\hbar\tau] = (J\,s)(s) = J\,s^2. \tag{40}$$

Since

$$J = kg\,m^2\,s^{-2}, \tag{41}$$

we have

$$J\,s^2 = kg\,m^2. \tag{42}$$

Therefore,

$$\left[\frac{\hbar\tau}{r^2}\right] = \frac{kg\,m^2}{m^2} = kg. \tag{43}$$

Thus, $\hbar\tau/r^2$ has exactly the dimensions of mass. We introduce the closure form

$$m = \frac{\hbar\tau}{r^2}. \tag{44}$$

Before the temporal–spatial closure condition is imposed, Eq. (44) is a dimensionally mass-like closure quantity. Once the QLM transport condition

$$r = c\tau$$

is imposed, it becomes the inertial rest mass associated with the closure mode.

7 Recovery of $E = mc^2$ from Temporal–Spatial Closure

The QLM temporal–spatial closure condition is

$$r = c\tau. \tag{45}$$

Substituting Eq. (45) into Eq. (44) gives

$$\begin{aligned} m &= \frac{\hbar\tau}{(c\tau)^2} \\ &= \frac{\hbar\tau}{c^2\tau^2} \\ &= \frac{\hbar}{c^2\tau}. \end{aligned} \tag{46}$$

Using

$$\omega = \frac{1}{\tau}, \tag{47}$$

we obtain

$$m = \frac{\hbar\omega}{c^2}. \tag{48}$$

Multiplying by c^2,

$$mc^2 = \hbar\omega. \tag{49}$$

Thus the inertial energy relation is recovered directly from the temporal–spatial closure condition and the QLM phase-action relation $E = \hbar\omega$.

The same result may be written as a reduced-Compton closure. From $r = c\tau$,

$$\tau = \frac{r}{c}. \tag{50}$$

Substitution into Eq. (44) gives

$$\begin{aligned} m &= \frac{\hbar(r/c)}{r^2} \\ &= \frac{\hbar}{rc}. \end{aligned} \tag{51}$$

Solving for r,

$$r = \frac{\hbar}{mc}. \tag{52}$$

Therefore, the temporal–spatial closure form is equivalent to the reduced-Compton closure relation when $r = c\tau$.

8 Electron Rest Closure

For the electron, the rest energy is

$$E_e = m_e c^2. \tag{53}$$

Using the QLM phase-energy relation $E = \hbar\omega$, the electron rest angular frequency is

$$\omega_e = \frac{m_e c^2}{\hbar}. \tag{54}$$

The electron proper-time closure period is

$$\tau_e = \frac{1}{\omega_e}. \tag{55}$$

Substituting Eq. (54) gives

$$\tau_e = \frac{\hbar}{m_e c^2}. \tag{56}$$

The electron spatial rest-closure radius is the reduced Compton wavelength:

$$r_e = \bar{\lambda}_e = \frac{\hbar}{m_e c}. \tag{57}$$

The temporal–spatial closure condition is then

$$\begin{aligned} \frac{r_e}{\tau_e} &= \frac{\hbar/(m_e c)}{\hbar/(m_e c^2)} \\ &= c. \end{aligned} \tag{58}$$

Therefore,

$$r_e = c\tau_e. \tag{59}$$

Applying the general closure form to the electron gives

$$\begin{aligned} \frac{\hbar\tau_e}{r_e^2} &= \frac{\hbar\left(\frac{\hbar}{m_e c^2}\right)}{\left(\frac{\hbar}{m_e c}\right)^2} \\ &= \frac{\frac{\hbar^2}{m_e c^2}}{\frac{\hbar^2}{m_e^2 c^2}} \\ &= m_e. \end{aligned} \tag{60}$$

Thus,

$$m_e = \frac{\hbar\tau_e}{r_e^2} = \frac{\hbar\tau_e}{\bar{\lambda}_e^2}, \qquad r_e = \bar{\lambda}_e = c\tau_e. \tag{61}$$

The electron is therefore a lower-frequency, larger-radius temporal–spatial closure mode relative to the saturated Planck mode. This statement does not determine why the electron has its measured mass; it shows that the measured electron rest scale fits the same closure structure once m_e is supplied.

9 Electron-to-Planck Mass Ratio

Define the electron-to-Planck mass ratio

$$\epsilon_e \equiv \frac{m_e}{m_P}. \tag{62}$$

Using

$$m_e = \frac{\hbar\tau_e}{r_e^2}, \qquad m_P = \frac{\hbar t_P}{\ell_P^2}, \tag{63}$$

we find

$$\begin{aligned}\frac{m_e}{m_P} &= \frac{\hbar\tau_e/r_e^2}{\hbar t_P/\ell_P^2} \\ &= \frac{\tau_e}{t_P}\left(\frac{\ell_P}{r_e}\right)^2. \end{aligned} \tag{64}$$

Using

$$r_e = c\tau_e, \qquad \ell_P = ct_P, \tag{65}$$

we obtain

$$\frac{\ell_P}{r_e} = \frac{ct_P}{c\tau_e} = \frac{t_P}{\tau_e}. \tag{66}$$

Substituting into Eq. (64),

$$\begin{aligned}\frac{m_e}{m_P} &= \frac{\tau_e}{t_P}\left(\frac{t_P}{\tau_e}\right)^2 \\ &= \frac{t_P}{\tau_e}. \end{aligned} \tag{67}$$

Since $\tau_e = 1/\omega_e$,

$$\frac{m_e}{m_P} = \omega_e t_P. \tag{68}$$

Using $r_e = c\tau_e$ and $\ell_P = ct_P$, the same ratio is

$$\frac{m_e}{m_P} = \frac{\ell_P}{r_e} = \frac{\ell_P}{\bar{\lambda}_e}. \tag{69}$$

Therefore,

$$\epsilon_e = \frac{m_e}{m_P} = \frac{t_P}{\tau_e} = \frac{\ell_P}{r_e} = \frac{\ell_P}{\bar{\lambda}_e} = \omega_e t_P. \tag{70}$$

This identity admits three equivalent interpretations:

(i) $\epsilon_e = t_P/\tau_e$ is the inverse temporal expansion of the electron rest-closure period relative to the Planck tick.

(ii) $\epsilon_e = \ell_P/r_e$ is the inverse spatial expansion of the electron closure radius relative to the Planck length.

(iii) $\epsilon_e = \omega_e t_P$ is the electron rest-phase advance per Planck tick.

Because

$$\tau_e \gg t_P, \qquad r_e \gg \ell_P, \qquad m_e \ll m_P, \tag{71}$$

the electron is a non-saturated closure mode.

10 Invariant Electron Mass Versus Routed Momentum

A key distinction must be preserved: routing suppression can alter transported momentum or externally observed phase-action flow without changing the intrinsic electron rest mass.

The local electron rest closure is

$$m_e c^2 = \hbar\omega_e, \qquad r_e = \bar{\lambda}_e = c\tau_e. \tag{72}$$

These are local proper-time relations. They define the invariant electron rest closure.

Momentum transport is described by

$$p_i = \hbar k_i. \tag{73}$$

The intrinsic electron rest momentum scale is

$$p_{e,\mathrm{rest}} = m_e c. \tag{74}$$

A routed momentum channel may be suppressed by a routing availability factor. Schematically,

$$p_{e,\mathrm{route}} = Y\, p_{e,\mathrm{local}}, \tag{75}$$

where Y is a dimensionless routing availability or admittance fraction.

For a gravitational routing field, define

$$Y_g(r) = 1 - \frac{r_s}{r}, \tag{76}$$

and

$$Z_g(r) = Y_g(r)^{-1/2} = \left(1 - \frac{r_s}{r}\right)^{-1/2}. \tag{77}$$

Then the momentum or energy transported to an asymptotic observer is throttled as

$$p_\infty = \sqrt{Y_g(r)}\, p_{\mathrm{local}} = \frac{p_{\mathrm{local}}}{Z_g(r)}, \tag{78}$$

and

$$E_\infty = \sqrt{Y_g(r)}\, E_{\mathrm{local}} = \frac{E_{\mathrm{local}}}{Z_g(r)}. \tag{79}$$

For an emitted photon,

$$\omega_\infty = \omega_{\mathrm{local}} \frac{d\tau}{dt_\infty} = \omega_{\mathrm{local}} \sqrt{Y_g(r)} = \frac{\omega_{\mathrm{local}}}{Z_g(r)}. \tag{80}$$

Thus,

$$E_\infty = \hbar\omega_\infty = \frac{\hbar\omega_{\mathrm{local}}}{Z_g(r)}. \tag{81}$$

The local electron rest mass remains

$$m_e c^2 = \hbar\omega_e. \tag{82}$$

Therefore, gravitational or geometric routing throttles transported momentum and outward phase-action flow; it does not change the electron's intrinsic rest closure mass.

11 Relation to Fine-Structure Routing

The temporal–spatial closure relation determines the intrinsic electron rest closure scale

$$r_e = \bar{\lambda}_e = \frac{\hbar}{m_e c}. \tag{83}$$

This scale belongs to the local electron rest-mass closure and is not itself a Coulomb-bound orbital radius.

In hydrogenic Coulomb routing, the available routed momentum channel is suppressed by the fine-structure fraction α. The orbital momentum scale is therefore

$$p_{e,\mathrm{orb}} = \alpha m_e c. \tag{84}$$

Using $p = \hbar/r$, the corresponding orbital radius is

$$\begin{aligned} a_0 &= \frac{\hbar}{p_{e,\mathrm{orb}}} \\ &= \frac{\hbar}{\alpha m_e c} \\ &= \frac{\bar{\lambda}_e}{\alpha}. \end{aligned} \tag{85}$$

Thus, α does not replace the electron rest closure scale. Instead, it controls how the already-defined electron closure scale is expanded into the Coulomb-routed orbital scale:

$$r_e = \bar{\lambda}_e \longrightarrow a_0 = \frac{\bar{\lambda}_e}{\alpha}. \tag{86}$$

In Planck-normalized form,

$$\frac{\bar{\lambda}_e}{\ell_P} = \epsilon_e^{-1}, \tag{87}$$

while

$$\frac{a_0}{\ell_P} = \frac{1}{\alpha \epsilon_e}. \tag{88}$$

The hydrogenic binding scale follows from the routed orbital momentum:

$$\begin{aligned} E_H &= \frac{p_{e,\mathrm{orb}}^2}{2m_e} \\ &= \frac{1}{2}\alpha^2 m_e c^2. \end{aligned} \tag{89}$$

Since

$$E_P = m_P c^2, \tag{90}$$

we obtain

$$\begin{aligned} \frac{E_H}{E_P} &= \frac{\frac{1}{2}\alpha^2 m_e c^2}{m_P c^2} \\ &= \frac{1}{2}\alpha^2 \frac{m_e}{m_P} \\ &= \frac{1}{2}\alpha^2 \epsilon_e. \end{aligned} \tag{91}$$

The hierarchy is therefore

$$\epsilon_e \longrightarrow \epsilon_e^{-1} \longrightarrow (\alpha\epsilon_e)^{-1} \longrightarrow \frac{1}{2}\alpha^2\epsilon_e. \tag{92}$$

These correspond respectively to

$$\frac{m_e}{m_P}, \qquad \frac{\bar{\lambda}_e}{\ell_P}, \qquad \frac{a_0}{\ell_P}, \qquad \frac{E_H}{E_P}. \tag{93}$$

Thus, in this paper, the fine-structure constant enters only as a routed-momentum fraction relating the electron's intrinsic rest closure scale to the Coulomb-bound orbital scale.

12 Numerical Scale Summary

Using CODATA recommended empirical values for the electron mass and Planck mass [5], the electron-to-Planck ratio is approximately

$$\epsilon_e = \frac{m_e}{m_P} \approx 4.19 \times 10^{-23}. \tag{94}$$

Therefore,

$$\frac{\tau_e}{t_P} = \epsilon_e^{-1} \approx 2.39 \times 10^{22}, \tag{95}$$

and

$$\frac{r_e}{\ell_P} = \frac{\bar{\lambda}_e}{\ell_P} = \epsilon_e^{-1} \approx 2.39 \times 10^{22}. \tag{96}$$

The electron rest-phase advance per Planck tick is

$$\omega_e t_P = \epsilon_e \approx 4.19 \times 10^{-23}. \tag{97}$$

The electron is therefore not close to Planck saturation. It is a vastly expanded temporal–spatial closure mode whose local rest-phase advance per Planck tick is extremely small.

13 Conclusion

This paper has shown that the temporal–spatial closure form

$$m = \frac{\hbar\tau}{r^2} \tag{98}$$

recovers

$$mc^2 = \hbar\omega \tag{99}$$

when the QLM transport condition $r = c\tau$ is imposed. The result gives a compact QLM route from local phase-action closure to the inertial energy relation $E = mc^2$.

The Planck and electron cases serve as scale-separated realizations of the same relation: the Planck case saturates the lattice tick and spacing, while the electron case occupies a much larger temporal and spatial closure scale. In both cases, mass is represented as local closure inventory rather than as routed momentum.

Within QLM, routed momentum and emitted phase-action signals may be suppressed by the available transport geometry without changing intrinsic rest mass. The fine-structure constant enters only as a Coulomb-routing fraction that maps the electron rest closure radius $\bar{\lambda}_e$ to the Bohr scale $a_0 = \bar{\lambda}_e/\alpha$.

The result is intentionally narrow: it does not determine the numerical electron mass or the fine-structure constant. It shows that Planck saturation, the general mass closure form, and the electron rest scale share one temporal–spatial closure structure.

A Notation and Symbol Definitions

Symbol / Notation	**Definition**
QLM primitives and lattice quantities	
$\hbar$	Reduced Planck constant; primitive reduced-action quantum of the QLM.
ℓ_P	Planck length; primitive lattice spatial increment.
t_P	Planck time; primitive lattice temporal increment or proper-time tick.
c	Invariant lattice transport speed, $c = \ell_P/t_P$.
E_P	Planck energy throughput, $E_P = \hbar/t_P$.
m_P	Planck mass, $m_P = \hbar t_P/\ell_P^2$.
ω_P	Planck angular frequency, $\omega_P = 1/t_P$.
Phase, action, and closure variables	
θ	Dimensionless physical phase measured in radians.
τ	Local proper-time closure period.
ω	Angular phase rate, $\omega = d\theta/d\tau$; for a closure period, $\omega = 1/\tau$.
E	Energy or phase-action throughput, $E = \hbar\omega$.
m	Rest mass of a temporal–spatial closure mode.
r	Spatial closure radius associated with the local proper-time closure period.
$m = \hbar\tau/r^2$	Temporal–spatial mass closure form studied in this paper.
$r = c\tau$	QLM temporal–spatial transport condition for the closure mode.
$mc^2 = \hbar\omega$	Inertial energy relation recovered from $m = \hbar\tau/r^2$ when $r = c\tau$.
Electron rest-closure quantities	
m_e	Electron rest mass.
E_e	Electron rest energy, $E_e = m_e c^2$.
ω_e	Electron rest angular frequency, $\omega_e = m_e c^2/\hbar$.
τ_e	Electron rest closure period, $\tau_e = 1/\omega_e = \hbar/(m_e c^2)$.
r_e	Electron spatial rest-closure radius.
$\bar{\lambda}_e$	Electron reduced Compton wavelength, $\bar{\lambda}_e = \hbar/(m_e c)$.
$r_e = \bar{\lambda}_e = c\tau_e$	Electron temporal–spatial rest-closure condition used in this paper.
ϵ_e	Electron-to-Planck mass ratio, $\epsilon_e = m_e/m_P = t_P/\tau_e = \ell_P/r_e = \omega_e t_P$.
Momentum and phase-gradient variables	
p_i	Momentum component.
k_i	Spatial phase-gradient component, with $p_i = \hbar k_i$.
$p_{e,\text{rest}}$	Intrinsic electron rest momentum scale, $p_{e,\text{rest}} = m_e c$.
$p_{e,\text{route}}$	Routed electron momentum through an available transport channel.
$p_{e,\text{orb}}$	Coulomb-routed orbital momentum scale, $p_{e,\text{orb}} = \alpha m_e c$.
Routing and gravitational quantities	
Y	Dimensionless routing availability or admittance fraction.
$Y_g(r)$	Gravitational routing availability, $Y_g(r) = 1 - r_s/r$.
$Z_g(r)$	Gravitational impedance or throttle factor, $Z_g(r) = Y_g(r)^{-1/2}$.
r_s	Schwarzschild radius.

Symbol / Notation	Definition
p_∞	Momentum transported to an asymptotic observer.
E_∞	Energy transported to an asymptotic observer.
ω_∞	Photon angular frequency measured by an asymptotic observer.
Fine-structure and hydrogenic quantities	
α	Fine-structure constant; interpreted here only as a Coulomb routing fraction.
a_0	Bohr radius, $a_0 = \bar{\lambda}_e/\alpha$.
E_H	Hydrogenic binding scale, $E_H = \frac{1}{2}\alpha^2 m_e c^2$.

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